Lecture 16

PID controller

Prof Peter YK Cheung

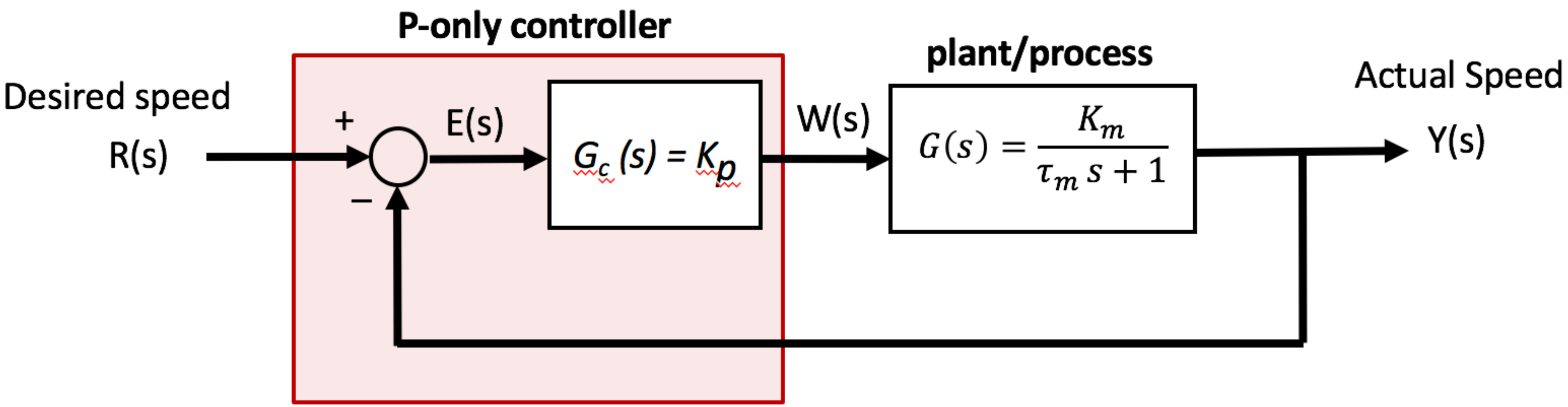
Dyson School of Design Engineering

URL: www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/
E-mail: p.cheung@imperial.ac.uk

Limitations of Proportional-only (P) control

- In **proportional-only** control, the controller output is given by: $w(t) = K_p e(t)$
- Using P-only control is simple, but often insufficient because:

1. If Kp is small, **error** e(t) can be large (i.e. there is an offset error between set-point and controlled output variable, in this case, speed of motor).
2. If Kp is large, the system may oscillate (i.e. become unstable).
3. Even if the system is stable, it may take a long time to settle to its final output value or exhibition large overshoots.
4. It may not have sufficient tolerance to perturbations or disturbances.



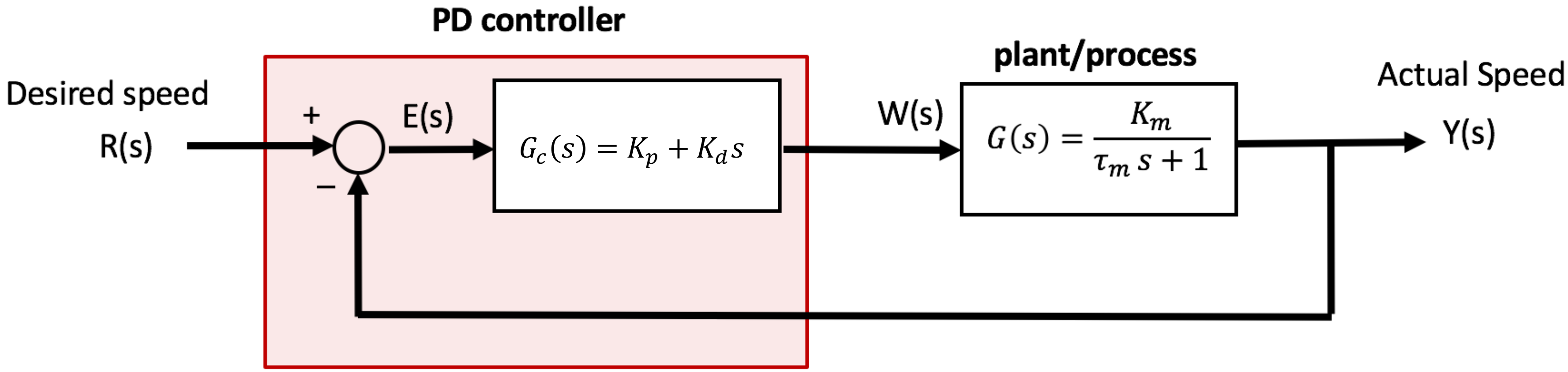
Proportional - Derivative (PD) Control

- We can add another term to include the rate of change of the error $\dot{e}(t)$. This is known as a **proportional-derivative** (PD) controller: $w(t) = K_p e(t) + K_d \dot{e}(t)$
- In computers, the **derivative term** $\dot{e}(t)$ is usually calculated by taking the **difference** between current error value e[n] and the previous error value e[n-1]:

$$\boxed{\text{differential term at time n} = K_d\ (e[n] - e[n-1])/\Delta t}$$

- The main advantages of the PD controllers are:

1. It can reduce the overshoot of a proportional-only controller response because PD controller takes into account the rate of change in error.
2. It can also improve the system's tolerance to external disturbances.



Proportional - Integral (PI) Control

- Alternatively, we can add an **integral term** to the controller. This is known as a PI controller:

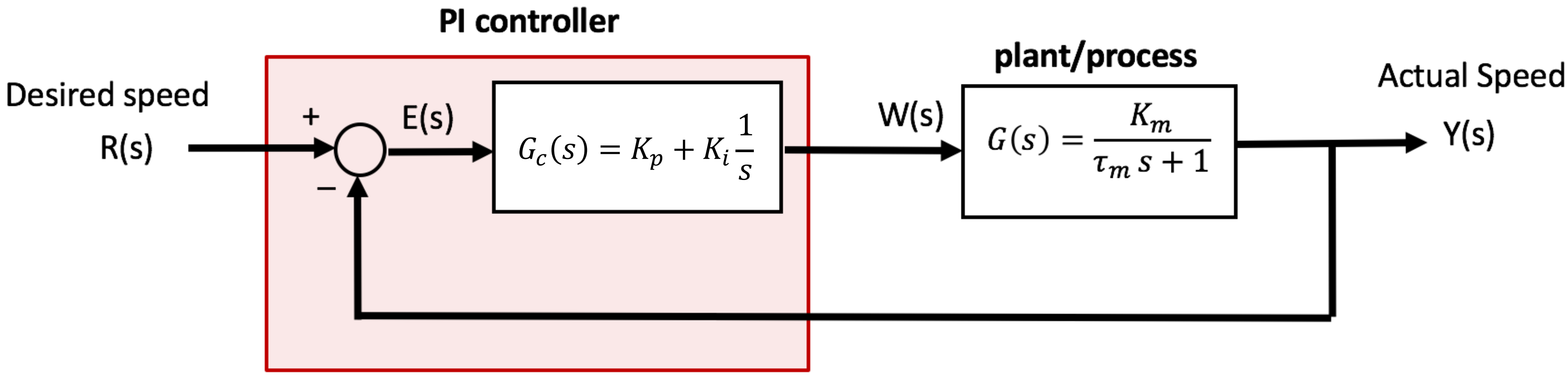
$$u(t) = K_p e(t) + K_i \int e(\tau)d\tau$$

- The integral term is implemented on a digital microprocessor as **summation** (Δt is the sampling interval):

$$\text{integral term at time n} = K_i \Delta t \sum_{k=0}^{n} e[n-k]$$

- The main advantages of the PI controller are:

1. It eliminates **steady-state error**.
2. It can help with **stability of the system**, especially if K_p is large.



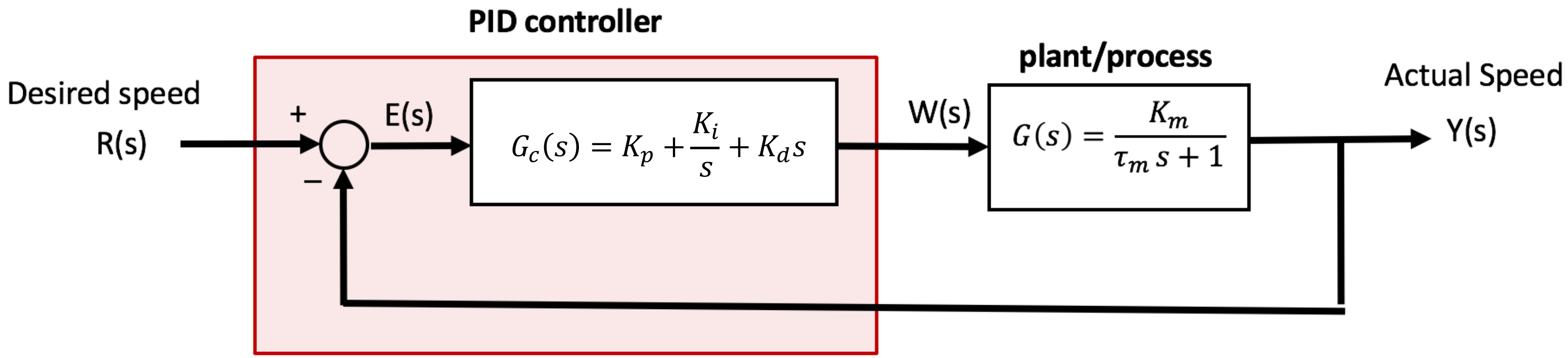
PID Control

- Finally, we can combine all three terms to form a PID controller:

$$u(t) = K_p e(t) + K_i \int e(\tau) d\tau + K_d \dot{e}(t)$$

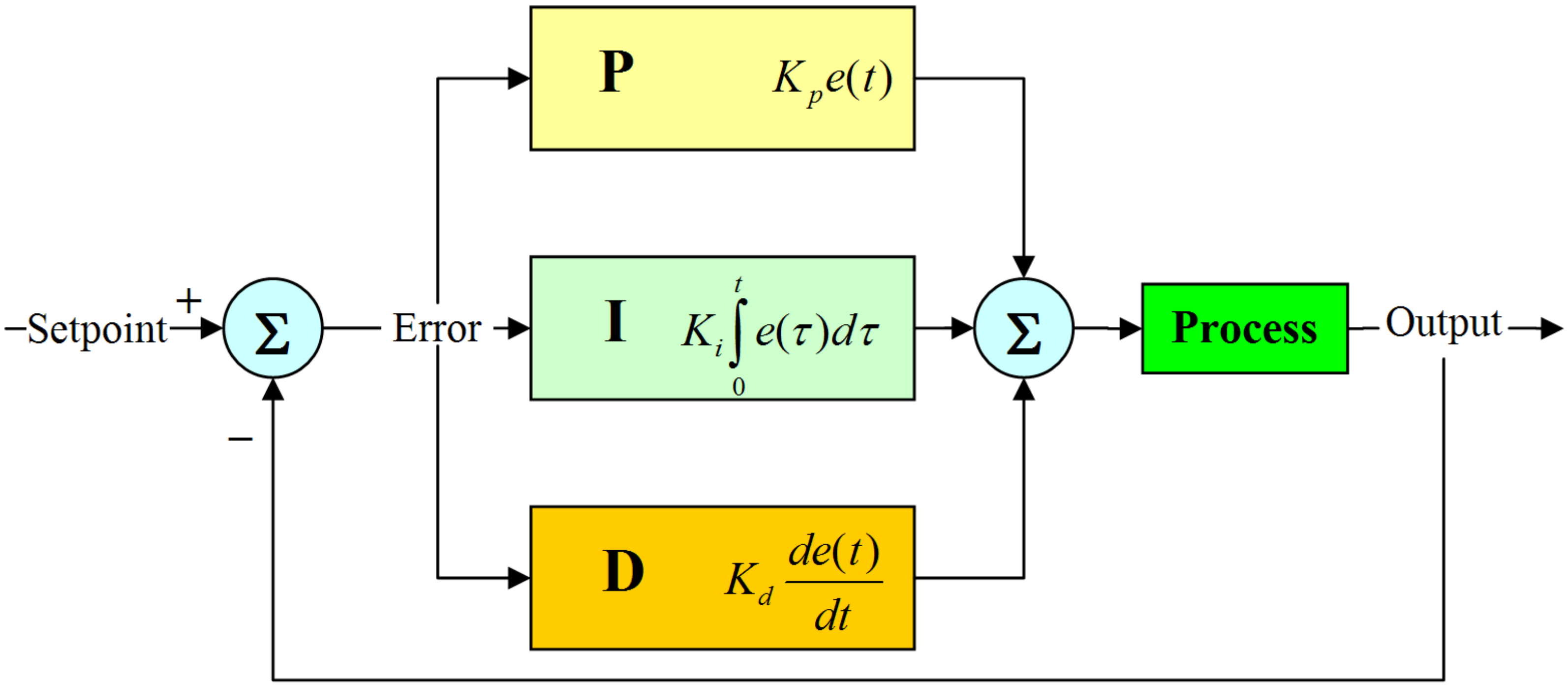
- This has the advantages of **ALL** three types of feedback control (P, I and D):

1. **Removal** of steady-state **error** due to I.
2. **Reduce** the amount of **overshoots** (due to be I and D).
3. **Improve** the **transient** response to make it faster (due to both I and D).
4. **Improve stability** of the system.
5. Better perturbation tolerance.

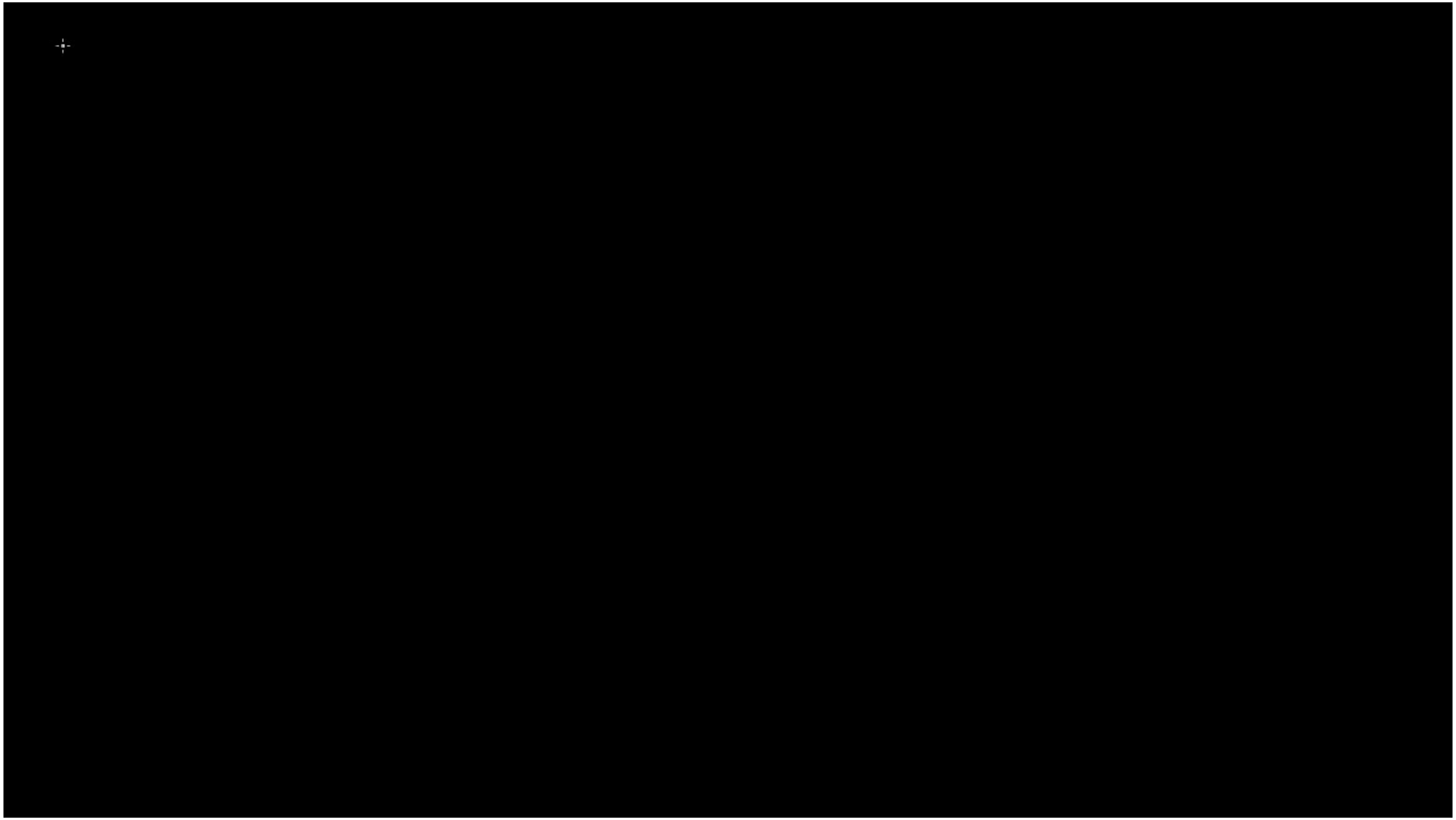


PID Controller block diagram

- Here is a schematic of an implementation of a typical PID controller.
- In practice, we often use PI (e.g. driving a motor), or PD (e.g. balancing two-wheel vehicle), and NOT all three terms.



Use PID controller to drive a car



Tuning of a PID Controller

- Choosing the correct values for K_p, K_d and K_i is known as **tuning** the controller.
- **Impact of** various **gains** on step response of a system:

PID Gain	Percent Overshoot	Settling Time	Steady-State Error
Increasing K_P	Increases	Minimal impact	Decreases
Increasing K_I	Increases	Increases	Zero steady-state error
Increasing K_D	Decreases	Decreases	No impact

- We will now consider two approaches to tuning the PID controller:

1. Ziegler-Nichols method
2. Trial-and-error manual tuning

Ziegler-Nichols method of tuning PID controller

1. Set K_d and K_i to **zero**.
2. Adjust K_p from 0 until the system starts to **oscillate** at certain frequency.
3. **Measure** the value $K_u = K_p$, and the oscillation period as T_u.
4. **Set** the various **gain factors** according to the follow formula and table:

$$u(t) = K_p(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\dot{e}(t))$$

Control Type	K_p	T_i	T_d
P	$0.5K_u$	-	-
PI	$0.45K_u$	$T_u/1.2$	-
PD	$0.8K_u$	-	$T_u/8$
classic PID	$0.6K_u$	$T_u/2$	$T_u/8$

Manual method of tuning PID controller

1. Set K_p , K_d and K_i to zero.
2. Start with a small K_p, double it each time until the system starts to oscillate.
3. Half the value of K_p.
4. Start with a small K_d, double it each time until the system starts to oscillate.
5. Half the value of K_d .
6. Start with a small K_i, double it each time until the system starts to oscillate.
7. Half the value of K_i.

- If you are only using PD or PI controller, skip the irrelevant steps.
- Fine tune the various gain until you get the response you want.

Segway balancing with PID controller

- For the team project, we need to balance the Segway using feedback control.
- Instead of using motor speed as the control variable, we should use the **pitch angle** as the **control variable**.
- Available for us to use is the **pitch angle** (after passing through a **complementary filter**) p and the **rate of change of pitch angle** (from gyroscope alone) $\dot{p}$.
- Since we have $\dot{p}$ available, the best controller to use is a PD controller, where the control variable is the pitch angle p.
- For **normal balancing** action, the **set-point r(t) is 0**, i.e. upright position.
- The controller output value w(t) is derived with all P, I and D terms of the pitch angle:

$$w(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int e(\tau) d\tau$$

- w(t) is used to drive the motors forward or backward in order to keep pitch angle p = 0, by producing PWM values to drive the two motors.
- To **move** the Segway forward or backward, **change the set-point** r(t) to some other values (a small positive or negative angle).
- To **turn** right or left, you need also to **adjust the ratio** of PWM duty cycle between the two motors.